Exploring the Use of Alfvén Waves in Magnetometer Calibration at Geosynchronous Orbit

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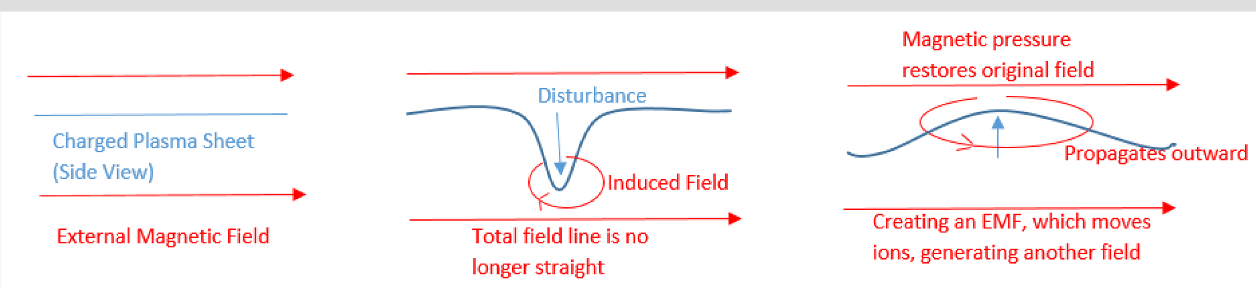
Abstract

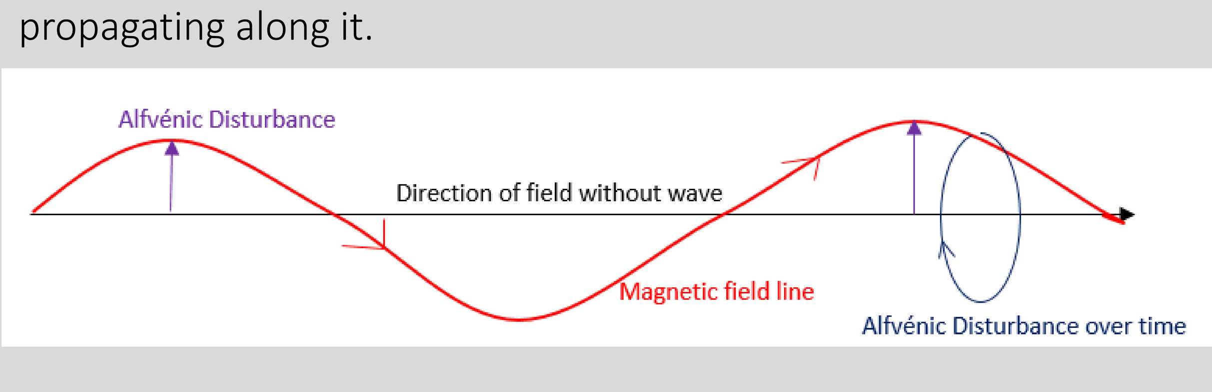
An Alfvén wave is a type magnetohydrodynamic wave that travels through a conducting fluid under the influence of a magnetic field. Researchers have successfully calculated offset vectors of magnetometers in interplanetary space by optimizing the offset to maximize certain Alfvénic properties of observed waves (Leinweber, Belcher). If suitable Alfvén waves can be found in the magnetosphere at geosynchronous altitude then these techniques could be used to augment the overall calibration plan for magnetometers in this region such as on the GOES spacecraft, possibly increasing the time between regular maneuvers. Calibration maneuvers may be undesirable because they disrupt the activities of other instruments. Various algorithms to calculate an offset using Alfvén waves were considered. A new variation of the Davis-Smith method was derived because it can be mathematically shown that the Davis-Smith method tolerates filtered data, which expands potential applications. The variant developed was designed to find only the offset in the plane normal to the main field because the overall direction of Earth's magnetic field rarely changes, and theory suggests the Alfvénic disturbances occur transverse to the main field. Other variations of the Davis-Smith method encounter problems with data containing waves that propagate in mostly the same direction. A searching algorithm was then designed to look for periods of time with potential Alfvén waves in GOES 15 data based on parameters requiring that disturbances be normal to the main field and not change field magnitude. Final waves for calculation were hand-selected. These waves produced credible two-dimensional offset vectors when input to the Davis-Smith method. Multiple two-dimensional solutions in different planes can be combined to get a measurement of the complete offset. The resulting three dimensional offset did not show sufficient precision over several years to be used as a primary calibration method, but reflected changes in the offset fairly well, suggesting that the method could be helpful in monitoring trends of the offset vector when maneuvers cannot be used.

Background on Alfvén Waves

Alfvén waves are an interaction between a charged plasma and a constant magnetic field. Hannes Alfvén first described his namesake waves in 1942 by deriving the wave equation from Maxwell's equations and the equations of fluid dynamics. A disturbance in the plasma causes the ions in the plasma to move in one direction simultaneously, which constitutes a current. The current in the plasma generates a magnetic field that distorts the total magnetic field, which now constitutes the external field plus the locally generated field. The magnetic pressure of the external field acts as the restoring force, straightening the field lines and therefore inducing a current in the plasma in the opposite direction of the original current. This new current generates another magnetic field which is again corrected by the magnetic pressure, generating a current in the original direction. Thus, a self-propagating magnetohydrodynamic wave is created. Below is a diagram of an Alfvén wave in a two-dimensional plasma sheet.



There are many kinds of Alfvén waves. In the interplanetary solar wind, the direction of the observed magnetic field is chaotic, while the magnitude is comparatively constant (Leinweber). If Alfvén waves that are nearly circularly polarized, such that total observed field magnitude is conserved, exist at geosynchronous altitude, then the premises required for Leinweber's application of the Davis-Smith are met and the method could be used to find magnetometer offsets. Circularly polarized, or torsional, Alfvén waves are characterized by the induced component of the observed magnetic field being orthogonal to the main field, and this component of the field moving circularly around the main field. Thus the magnetic field lines in the solar wind move circularly around the vector of the field's overall direction when under the influence of a torsional Alfvén wave. These waves have the property that the magnitude of the total field does not change, only its direction. Below is a diagram showing a section of a magnetic field line in the solar wind with a torsional wave propagating along it.



Calibration Using Alfvén Waves

Torsional Alfvén waves do not change field magnitude and consist of sinusoidal disturbances orthogonal to the main field. The David-Smith method is an equation that returns the zero-offset that, when subtracted from the data, minimizes variance in the field magnitude. In other words, the equation maximizes the Alfvénic character of an observed wave. It is important to input windows that likely do contain Alfvén waves with relatively little noise. The equation is derived by finding the minimum magnitude variance over a constant offset vector and is reproduced below. Because the equation is entirely covariances, any constant value can be added or subtracted to the data without changing the result. This allows filtered data to be given to the equation, as long as the filter is independently applied to each component and then to the magnitude squared, as removing the average value of the field with a high-pass filter will not change results.

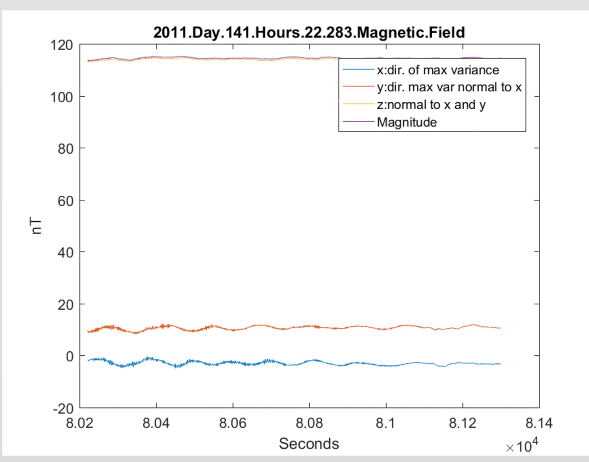
The Davis-Smith Equation

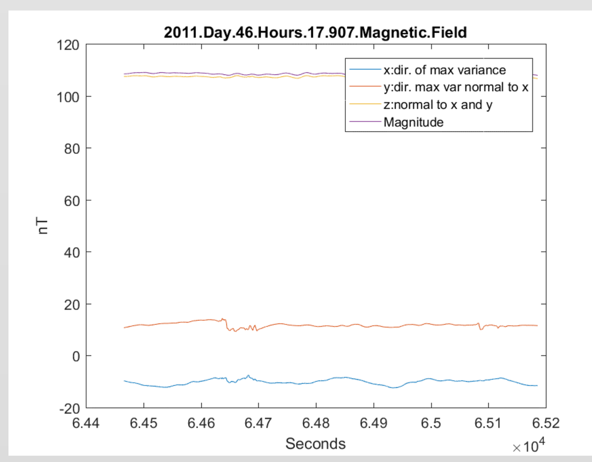
$$\begin{pmatrix} var(x) & cov(x,y) & cov(x,z) \\ cov(y,x) & var(y) & cov(y,z) \\ cov(z,x) & cov(z,y) & var(z) \end{pmatrix} \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} cov(x,B_M^2) \\ cov(y,B_M^2) \\ cov(z,B_M^2) \end{pmatrix}$$

Since there are Alfvénic disturbances only in the plane orthogonal to the main field, however, it is misguided to try to find a three-dimensional offset vector from just one wave. Leinweber proposes a way to combine several waves in different planes and then solve for the offset. This method seems to encounter problems if almost all of the wave point in the same direction. Because Earth's field largely does not change direction, a two-dimensional version of the equation was derived to solve for only the offset in the plane of Alfvénic disturbance. This offset was then transformed back into the standard basis. If several two-dimensional offsets are found in different planes, the full offset can be found by doing a search for the offset that minimizes the sum of the squared difference between each planar offset and the total offset when projected into the plane of the two-dimensional offset.

Finding Alfvén Waves

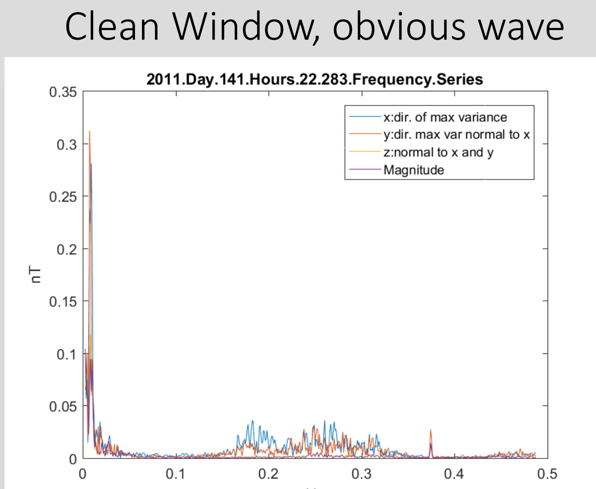
There is much more non-Alfvénic noise in the magnetosphere than in the solar wind. A year was searched for windows that met loose criteria suggesting that components exhibited more variance than the magnitude and that disturbances were normal to the main field. The selection parameters were made as strict as possible to get the about fifty cleanest potential Alfvén waves that last for ten minutes or longer. GOES-15 data from 2011 was used, because it is expected that the offset is well-known and corrected for during the first year of operations. Before filtering, some windows are chaotic and some are promising.

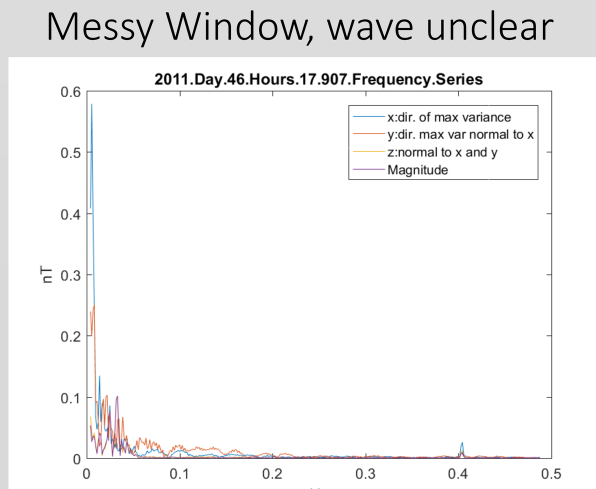




Clean Window, obvious wave

Messy Window, wave unclear





A Fourier transform with a periodic Hamming window was used to help identify the range of frequencies where the two components normal to the main field vary and the magnitude does not. Waves seem to be primarily in the Pc5 frequency range. If such frequencies cannot be found, the window may be omitted from calculation. It is up to the operator to determine the quality of waves. There are cases where having more windows may be better than having a small number of especially clean windows.

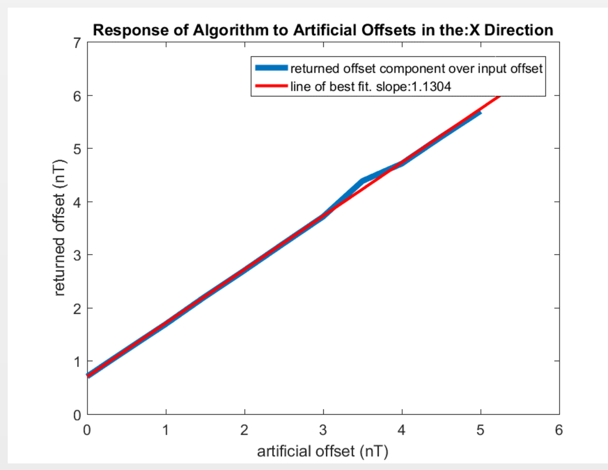
Filtering data at the frequencies of the suspected Alfvén waves was investigated, but proved unhelpful. Window lengths are short compared to the period of most of the Alfvén waves. Therefore, an FIR filter could not be implemented and IIR filters tended to be unstable, distorting the ends of the windows and allowing aliasing. For this exploration, it was decided to use unfiltered data, and rely on combining large numbers of windows into the calculation of the final offset to reduce the effect of noise.

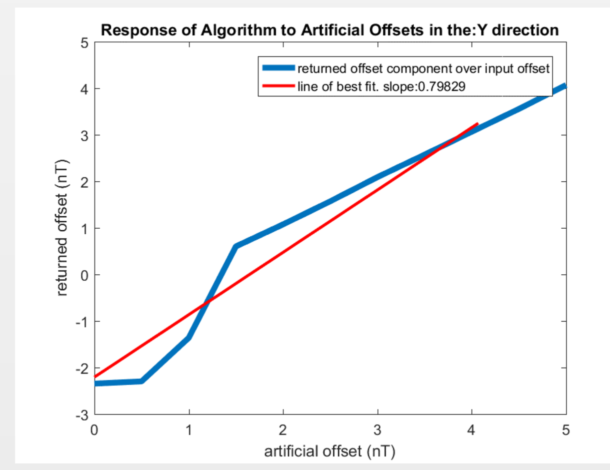
Testing the Algorithm and Results

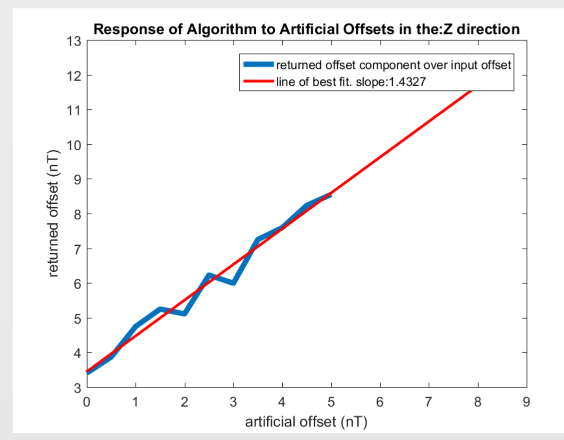
The effectiveness of the two-dimensional variant of the Davis-Smith Method was tested in three different ways. All windows that passed selection criteria were used for testing, with the belief that noisy windows would average out to zero. Results could potentially be improved by hand-selecting windows.

First, the overall offset was calculated for the GOES 15 inboard magnetometer in 2011. This alone does not provide very much information. The offset is expected to be credibly small. The returned offset using all windows that passed criteria was **(0.7066 -2.3467 3.4037)** nT in spacecraft body coordinates. This offset vector has a length of about **4** nT. This is somewhat outside the expected offset range for 2011, but believable.

Next, the response of the algorithm to changes in the offset was testing using artificial offsets. For each component, various offsets in only the component direction were added to the data, the total offset was computed using the two-dimensional Davis-Smith method plus offset combination, and the corresponding component of the returned offset was plotted against the known added offset. If the method is accurate, these plot should be linear with a slope of one, indicating that changes in the actual offset are accurately reflected by changes to the returned offset.

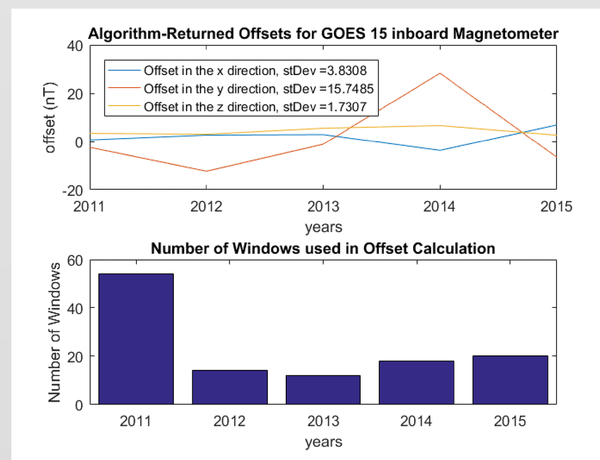






These graphs show that the algorithm is quite effective at mapping overall trends in the offset. With knowledge of the initial offset, the ability to roughly track changes in the offset vector could be very useful to spacecraft operators.

Finally, the total offset was calculated for GOES 15 for each year from 2011 to 2015. The offset was expected to undergo a random walk of .2 nT / year. Therefore, the standard deviation of each component over the five years should be comparable to 1nT.



It is not surprising that the offset in the y-direction is the most difficult to determine. The field is often nearly entirely in the y direction, meaning that for each window, much less information is obtained about the offset in the y direction that in the other directions. Fewer windows passed selection criteria for other years, which may have contributed to the low precision in the y component. Other components are closer to varying within the expected range. This suggests better precision for the x and z components of the offset.

Conclusion

The algorithm alone is not sufficient to independently determine the magnetometer offset in a given year. This may be because sufficiently circularly polarized Alfvén waves do not occur often enough at geosynchonous altitude for the method to be successful. The fact that the algorithm tracked artificial offsets fairly well illustrates that the algorithm is most likely sound, but that the input data is not clean enough, as the offset varied significantly more than expected. Varying selection parameters failed to isolate enough sufficiently circularly polarized Alfvén waves, but perhaps better filtering techniques or selection criteria could improve results. In the data, many potential linearly polarized Alfvén waves were observed. Therefore, continuing research will involve attempting to use other algorithms besides Leinweber's that use linearly polarized Alfvén waves to calculate parts of the offset vector.

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